



MINISTERUL  
EDUCAȚIEI



**Barem de notare și evaluare**  
**Olimpiada Națională de Matematică**  
**Etapa Locală, Județul Dolj, 17 februarie 2024**  
**clasa a VII a**

<b>Subiectul 1.</b>	
a) $\sqrt{70 + 30\sqrt{5}} = 5 + 3\sqrt{5}$	1p
$8\sqrt{7 + 3\sqrt{5}} = 4(3\sqrt{2} + \sqrt{10})$	1p
$\sqrt{7 - 3\sqrt{5}} - \sqrt{7 + 3\sqrt{5}} = \left(-\frac{\sqrt{10}}{2}\right).$	1p
$\frac{\sqrt{70+30\sqrt{5}}}{8\sqrt{7+3\sqrt{5}}} \cdot (\sqrt{7 - 3\sqrt{5}} - \sqrt{7 + 3\sqrt{5}}) = -\frac{5+3\sqrt{5}}{4(3\sqrt{2}+\sqrt{10})} \cdot \sqrt{10}$	1p
$-\frac{5+3\sqrt{5}}{4(3\sqrt{2}+\sqrt{10})} \cdot \sqrt{10} = -\frac{5 \cdot (5+3\sqrt{5})}{4 \cdot (3+\sqrt{5})} = -\frac{5}{4} \in \mathbb{Q}.$	1p
b) $[n] = \left[-\frac{5}{4}\right] = \left[-1\frac{1}{4}\right] = -2$	1p
$\{n\} = \left\{-1\frac{1}{4}\right\} = -1\frac{1}{4} - (-2) = \frac{3}{4}.$	1p
<b>TOTAL</b>	<b>7p</b>
<b>Subiectul 2</b>	
a) Folosind relația $ x ^2 = x^2 \Rightarrow$	
$ x - y ^2 +  x + y ^2 = x^2 - 2xy + y^2 + x^2 + 2xy + y^2 = 2x^2 + 2y^2$	1p
$2( x ^2 +  y ^2) = 2(x^2 + y^2) = 2x^2 + 2y^2$	
Finalizare : $ x - y ^2 +  x + y ^2 = 2( x ^2 +  y ^2)$	1p
b) Ridicând la pătrat obținem: $ x + y  +  x - y  =  x  +  y  +   x  -  y   \uparrow^2$	1p
$( x + y  +  x - y )^2 = [( x  +  y ) +   x  -  y  ]^2 \xleftrightarrow{ x ^2=x^2} x^2 + y^2 + 2xy + x^2 + y^2 - 2xy + 2 x^2 - y^2  = x^2 + y^2 + 2 xy  + x^2 + y^2 - 2 xy  + 2 x^2 - y^2 $	3p
Finalizare: $2 x^2 - y^2  = 2 x^2 - y^2 $	1p
<b>TOTAL</b>	<b>7p</b>

<b>Subiectul 3</b>	
a) $\triangle DTC$ – isoscel $\Rightarrow m(\angle CTD) = \frac{180^\circ - 30^\circ}{2} = 75^\circ$	1p
$\triangle ATE$ – isoscel $\Rightarrow m(\angle ATE) = \frac{180^\circ - (30^\circ + 60^\circ)}{2} = 45^\circ$	1p
Astfel $m(\angle ETC) = 180^\circ \Rightarrow C, T, E$ – coliniare	1p
b) $\begin{array}{l} \triangle ADE \\ \triangle CDF \end{array} \left  \begin{array}{l} DA = DC \\ ED = FD \\ \angle EDA = \angle FDC = 15^\circ \end{array} \right  \xrightarrow{L.U.L.} \triangle ADE \equiv \triangle CDF$	1p
Rezultă astfel $FC = AE$ și $m(\angle DCF) = 150^\circ \Rightarrow \angle FCB = 60^\circ$	1p
Dar $\triangle FBC$ – isoscel $\Rightarrow \triangle FBC$ – echilateral $\Rightarrow FB = BE \Rightarrow \triangle EBF$ – isoscel $\Rightarrow$	1p
$m(\angle EBF) = 150^\circ \Rightarrow m(\angle BFE) = 15^\circ$	1p
<b>TOTAL</b>	<b>7p</b>
<b>Subiectul 4</b>	
a) Fie $PO \cap DQ = \{R\}, OQ \cap AD = \{S\}$ . $\triangle OMA \equiv \triangle OBC, \triangle ONC \equiv \triangle OSA, \triangle ORS \equiv \triangle OMN \Rightarrow$ $OR = \frac{1}{2}PO, OS = \frac{1}{2}OQ, MN \equiv RS.$ $MN \parallel RS \Rightarrow RS \parallel PQ$ și $RS = \frac{PQ}{2}.$ în $\triangle DPQ \Rightarrow \left. \begin{array}{l} R - \text{mij.}(DQ) \\ S - \text{mij.}(PD) \end{array} \right\} \Rightarrow O - \text{c. g. } \triangle DPQ$	1p 1p 1p
b) Fie $DO \cap PQ = \{T\}$ $O - \text{c. g. } \triangle DPQ, OT = \frac{OD}{2} = \frac{OB}{2} = TB, \text{ dar } TQ = TP \Rightarrow OPBQ \text{ paralelogram}$	1p
c) " $\Rightarrow$ " $BP \equiv BQ \Rightarrow PBQO$ – romb $\Rightarrow \angle OTQ = 90^\circ, DT$ – mediană $\Rightarrow \triangle DPQ$ – is. $\triangle APB \equiv \triangle CQB \Rightarrow AP \equiv CQ \Rightarrow DA \equiv DC \Rightarrow ABCD$ – romb. " $\Leftarrow$ " $DT$ – mediană, $DT$ – bisectoarea $\angle PDQ \Rightarrow \triangle DPQ$ – isoscel $\Rightarrow DB \perp PQ \Rightarrow OB \perp PQ,$ dar $OBQO$ – paralelogram $\Rightarrow OBQO$ – romb $\Rightarrow BP \equiv BQ$	1p 1p 1p
<b>TOTAL</b>	<b>7p</b>